PROGRESSION THROUGH CALCULATIONS FOR MULTIPLICATION

We acknowledge Lancashire County Council as the main source for this document with some amendments specific to the Horwich and Blackrod Cluster .

It is imperative that children gain concrete experiences through play and manipulation before moving to any structured work or method. This is essential when any new concept is introduced.

- These standards are age-related expectations and therefore we expect the majority of children to achieve them.
- New learning is likely to be taught to groups rather than the whole class to acknowledge the different learning stages of the children.
- Children need to understand that multiplication is commutative and use this information to rearrange calculations knowing that 4 x 6 = 24 gives the same answer as 6 x 4 = 24. However increasingly they need to understand the context. (Working 6 hours at £4 p.h. = £24 and working 4 hours at £6 p.h. = £24 but a different outcome).
- Children need to understand that multiplication is repeated addition.
- Ensure that children understand the = sign means is the same as, not makes, and that children see calculations where the equals sign is in a different position, e.g. 3 x 5 = 15 and 15 = 3 x 5.
- Children should be encouraged to approximate before calculating and check whether their answer is reasonable.

By the end of year 6, children will have a range of calculation methods, mental and written. Selection will depend upon the numbers involved. Speed and competency is required for them to succeed.

Children should not be made to go onto the next stage if:

1) **they are not ready.**
2) **they are not confident. However written methods should be shown alongside other methods to help this to be achieved effectively.**

Children should be encouraged to consider if a mental calculation would be appropriate before using written methods.

Progression Towards a Written Method for Multiplication

In developing a written method for multiplication, it is important that children understand the concept of multiplication, in that it is:
- repeated addition

They should also be familiar with the fact that it can be represented as an array

They also need to understand and work with certain principles, i.e. that it is:
- the inverse of division
- commutative i.e. 5 x 3 is the same as 3 x 5

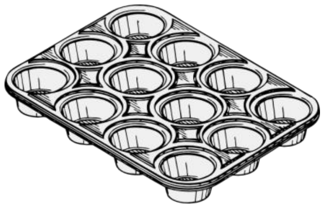
- associative i.e. 2 x 3 x 5 is the same as 2 x (3 x 5)

Reception

Early Learning Goal:
Children solve problems, including doubling.

Children are encouraged to develop a mental picture of the number system in their heads to use for calculation. They should experience practical calculation opportunities involving equal sets or groups using a wide variety of equipment, e.g. small world play, role play, counters, cubes etc. They develop ways of recording calculations using pictures, etc.

Children may also investigate putting items into resources such as egg boxes, ice cube trays and baking tins which are arrays.



They may develop ways of recording calculations using pictures, etc.

A child's jotting showing the fingers on each hand as a double.

A child's jotting showing double three as three cookies on each plate.

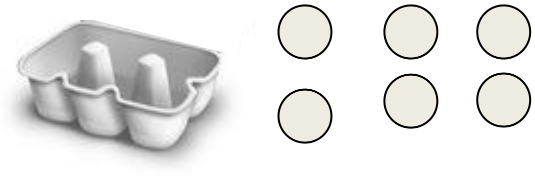
Resources: small world play, role play, counters, cubes, pairs of socks/gloves etc, models and images

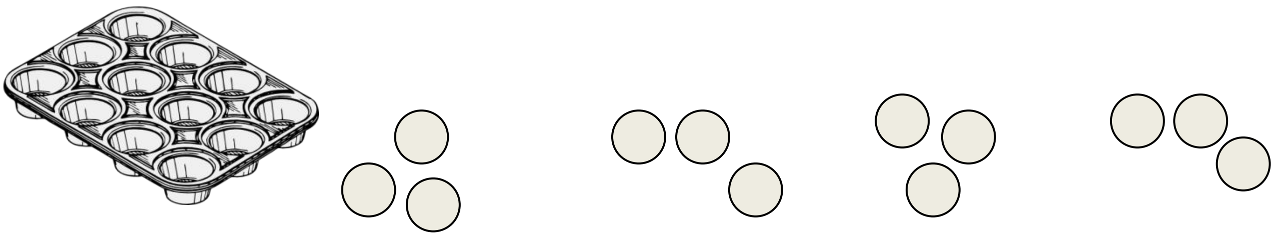
Key Vocabulary: equal sets or groups, counting, how many groups of the same number, counting in twos, pairs of

Y1

End of Year Objective:
Solve one-step problems involving multiplication by calculating the answer using concrete objects, pictorial representations and arrays with the support of the teacher.

Children will use practical equipment to make groups of objects to represent multiplication. They should see everyday versions of arrays, e.g. egg boxes, baking trays, ice cube trays, wrapping paper etc and use this in their learning answering questions such as 'How many eggs would we need to fill the egg box? How do you know?'





Resources: egg boxes, baking trays, ice cube trays, wrapping paper, models and images

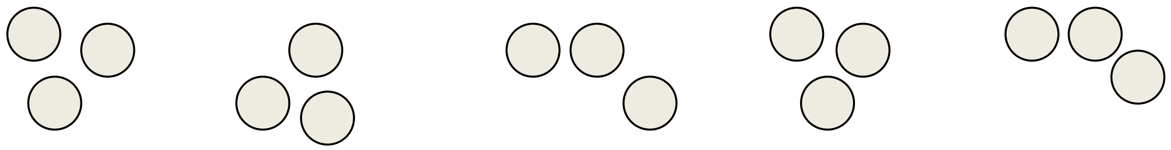
Key Vocabulary: Array, groups of, count on, count in twos, fives and tens, lots of

Y2

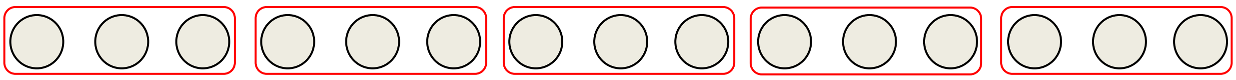
End of Year Objective:
Calculate mathematical statements for multiplication *(using repeated addition)* and write them using the multiplication (x) and equals (=) signs.

Children should understand and be able to calculate multiplication as repeated addition, supported by the use of practical apparatus such as counters or cubes. e.g.

5 x 3 can be shown as five groups of three with counters, either grouped in a random pattern, as below:

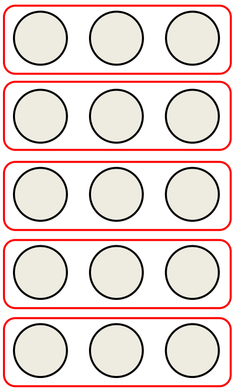


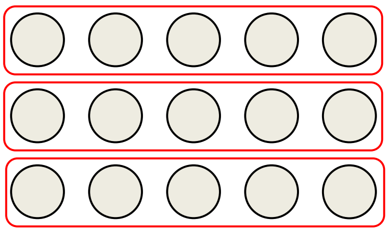
or in a more ordered pattern, with the groups of three indicated by the border outline:



Children should then develop this knowledge to show how multiplication calculations can be represented by an array, (this knowledge will support with the development of the grid method in the future). Again, children should be encouraged to use practical apparatus and jottings to support their understanding, e.g.

5 x 3* can be represented as an array in two forms (as it has commutativity):





3 + 3 + 3 + 3 + 3 = 15

3 x 5 = 15 is 5 lots of 3

5 + 5 + 5 = 15

5 x 3 = 15 is 3 lots of 5 = 15

*For mathematical accuracy 5 x 3 is represented by the second example above, rather than the first as it is five, three times. However, because we use terms such as 'groups of' or 'lots of', children are more familiar with the initial notation. (Once children understand the commutative order of multiplication the order is irrelevant).

Resources: concrete things if still required (see Yr1), number grids, whiteboards ITP arrays, models and images

Key Vocabulary: Lots of, groups of, multiply, x, multiply by, multiple of,, once, twice , three times etc, repeated addition array, row, column, double, group in pairs, threes, tens etc

Y3

End of Year Objective:
Write and calculate mathematical statements for multiplication using the multiplication tables that they know, including for two-digit numbers times one-digit numbers, progressing to formal written methods.*

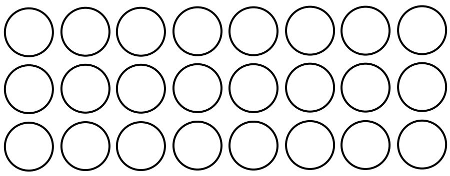
**Although the objective suggests that children should be using formal written methods, the National Curriculum document states "The programmes of study for mathematics are set out year-by-year for key stages 1 and 2. Schools are, however, only required to teach the relevant programme of study by the end of the key stage. Within each key stage, schools therefore have the flexibility to introduce content earlier or later than set out in the programme of study." p4*

It is more beneficial for children's understanding to go through the expanded methods of calculation as steps of development towards a formal written method.

Initially, children will continue to use arrays where appropriate linked to the multiplication tables that they know (2, 3, 4, 5, 8 and 10), e.g.

8 x 3

They may show this using practical equipment:



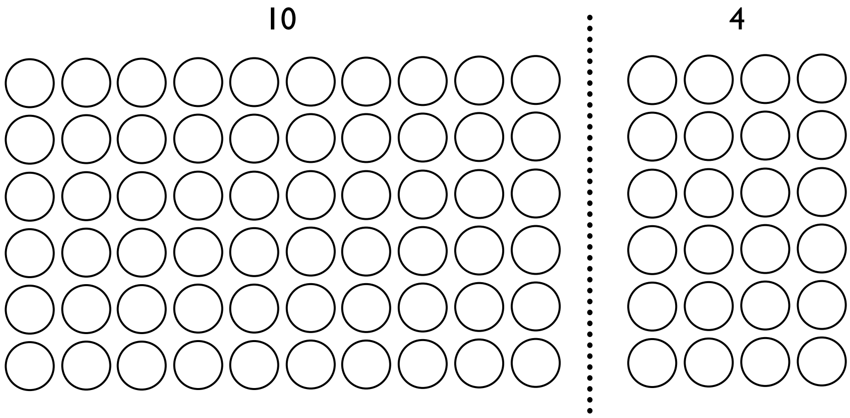
8 x 3 = 8 + 8 + 8 = 24

or by jottings using squared paper:

	x	x	x	x	x	x	x	x	
	x	x	x	x	x	x	x	x	
	x	x	x	x	x	x	x	x	

8 x 3 = 8 + 8 + 8 = 24

As they progress to multiplying a two-digit number by a single digit number, children should use their knowledge of partitioning two digit numbers into tens and ones to help them. For example, when calculating 14 x 6, children should set out the array, then partition the array so that one array has ten columns and the other four.



Partitioning in this way, allows children to identify that the first array shows 10 x 6 and the second array shows 4 x 6. These can then be added to calculate the answer:

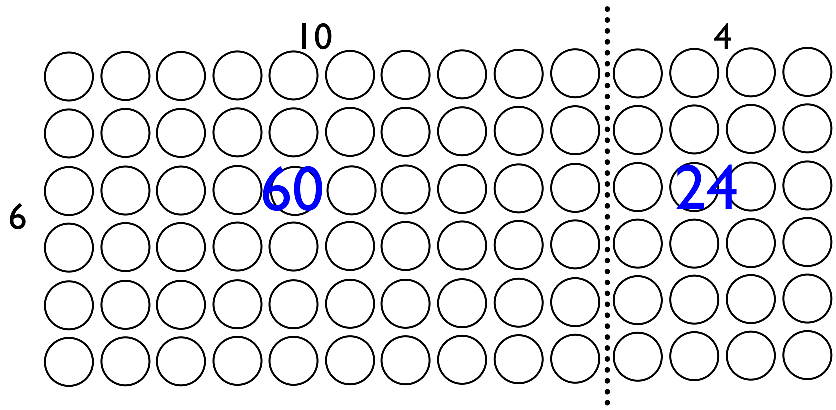
(6 x 10) + (6 x 4)

= 60 + 24

= 84

NB There is no requirement for children to record in this way, but it could be used as a jotting to support development if needed.

This method is the precursor step to the grid method. Using a two-digit by single digit array, they can partition as above, identifying the number of rows and the number of columns each side of the partition line.



By placing a box around the array, as in the example below, and by removing the array, the grid method can be seen.

x	10	4
6	60	24

It is really important that children are confident with representing multiplication statements as arrays and understand the rows and columns structure before they develop the written method of recording.

From this, children can use the grid method to calculate two-digit by one-digit multiplication calculations, initially with two digit numbers less than 20. Children should be encouraged to set out their addition in a column at the side to ensure the place value is maintained. When children are working with numbers where they can confidently and correctly calculate the addition mentally, (or supported by jottings or a number line) they may do so.

13 x 8

x	10	3
8	80	24

$$\begin{array}{r} 80 \\ + \quad 24 \\ \hline 104 \\ \hline \end{array}$$

When children are ready, they can then progress to using this method with other two-digit numbers.

37 x 6

x	30	7
6	180	42

$$\begin{array}{r} \text{HTO} \\ 180 \\ + \quad 42 \\ \hline 222 \\ \hline \end{array}$$

Children should also be using this method to solve problems and multiply numbers in the context of money or measures.

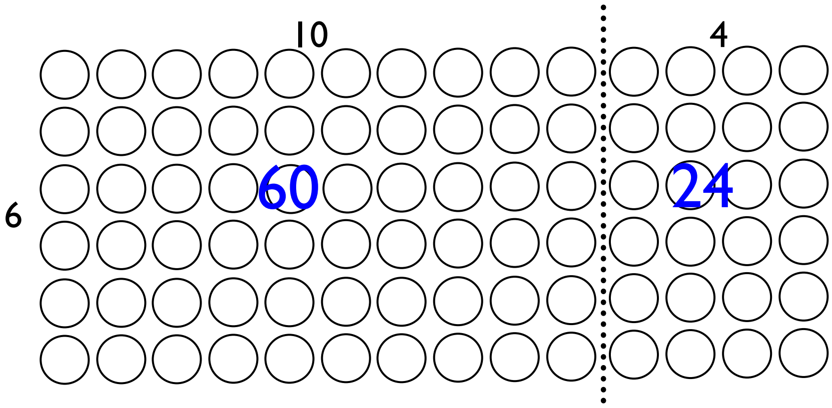
Resources: Practical resources as in Y2,

Key Vocabulary: product, multiplication, lots of, groups of, multiply, x, multiply by, multiple of, once, twice , three times etc, repeated addition array, row, column, double, group in pairs, threes, tens etc

Y4

End of Year Objective:
Multiply two-digit and three-digit numbers by a one-digit number using formal written layout.

Children will continue to use arrays where appropriate leading into the grid method of multiplication.



Partitioning in this way, allows children to identify that the first array shows 10 x 6 and the second array shows 4 x 6. These can then be added to calculate the answer:

(6 x 10) + (6 x 4)

= 60 + 24

= 84

NB There is no requirement for children to record in this way, but it could be used as a jotting to support development if needed.

TO (tens & ones)**x O** (Short multiplication – multiplication by a single digit)

23 x 8

x	20	3
8	160	24

```
  23
X  8
  24 (8X3)
 160 (8X20)
 184
```

HTO x O (Short multiplication – multiplication by a single digit)
346 x 9

x	300	40	6
9	2700	360	54

$$\begin{array}{r} 2700 \\ +\ 360 \\ \underline{+\ \ 54} \\ \underline{\mathbf{3114}} \end{array}$$

$$\begin{array}{rl} 346 & \\ \text{X}\ \underline{9} & \\ 54 & (9\text{x}6) \\ 360 & (9\text{X}40) \\ \underline{2700} & (9\text{X}300) \\ \underline{31\,14} & \\ {\scriptstyle 1\,1}\quad & \end{array}$$

The digits are carried at the bottom to mirror the addition method.

Children should build on their understanding and quickly move onto the formal written method shown in Year 5 as soon as they are ready.

Resources: counters,

Key Vocabulary: product, multiplication, lots of, groups of, multiply, x, multiply by, multiple of, once, twice , three times etc, repeated addition array, row, column, double, group in pairs, threes, tens etc

Y5

End of Year Objective:
Multiply numbers up to 4 digits by a one- or two-digit number using a formal written method, including long multiplication for two-digit numbers.

Children who are not confident with the short formal written method should continue to use the grid method and extend it to multiplying numbers with up to four digits by a single digit number, e.g.

4346 x 8

x	4 000	300	40	6
8	32 000	2400	320	48

```
  32000
+  2400
+   320
+    48
  34768
```

and numbers with up to four digits by a two-digit number, e.g.

2693 x 24

x	2000	600	90	3
20	40000	12000	1800	60
4	8000	2400	360	12

```
  40000
+  8000
+ 12000
+  2400
+  1800
+   360
+    60
+    12
  64632
```

When children are working with numbers where they can confidently and correctly calculate the addition (or parts of the addition) mentally, they may do so.

Children should also be using this method to solve problems and multiply numbers in the context of money or measures.

A decimal example:
4.9 X 3

x	4	0.9
3	12	2.7

```
   12.0
+   2.7
   14.7
```

$$\begin{array}{lr} & 4.9 \\ \text{X} & \underline{\;3} \\ & 2.7 \text{ (3X0.9)} \\ & \underline{12.0\ } \text{(3X4.0)} \\ & \underline{14.7} \end{array}$$

Only when mathematically ready, children should progress onto the formal algorithm (HTOxTO) Children in Year 6 are expected to use the short or long multiplication methods.

Extend to apply to money. 4.9 x 3 = £4.90   £4 x 3= £12
90p x 3= <u>£ 2.70</u>
£ 14.70

Short multiplication

24×6 becomes

$$\begin{array}{cccc} & & 2 & 4 \\ \times & & & 6 \\ \hline & 1 & 4 & 4 \\ \hline & & {\scriptstyle 2} & \end{array}$$

Answer: 144

342×7 becomes

$$\begin{array}{ccccc} & & 3 & 4 & 2 \\ \times & & & & 7 \\ \hline & 2 & 3 & 9 & 4 \\ \hline & & {\scriptstyle 2} & {\scriptstyle 1} & \end{array}$$

Answer: 2394

2741×6 becomes

$$\begin{array}{cccccc} & & 2 & 7 & 4 & 1 \\ \times & & & & & 6 \\ \hline & 1 & 6 & 4 & 4 & 6 \\ \hline & & {\scriptstyle 4} & {\scriptstyle 2} & & \end{array}$$

Answer: 16 446

Long multiplication

124×26 becomes

$$\begin{array}{cccc} & {\scriptstyle 1} & {\scriptstyle 2} & \\ & 1 & 2 & 4 \\ \times & & 2 & 6 \\ \hline & 7 & 4 & 4 \\ 2 & 4 & 8 & 0 \\ \hline 3 & 2 & 2 & 4 \\ \hline {\scriptstyle 1} & {\scriptstyle 1} & & \end{array}$$

Answer: 3224

(NC 2014)

Resources: whiteboards, ITP

Key Vocabulary: product, multiplication, lots of, groups of, multiply, x, multiply by, multiple of, once, twice , three times etc, repeated addition array, row, column, double, group in pairs, threes, tens etc

Y6

End of Year Objective:
Multiply multi-digit numbers up to 4 digits by a two-digit whole number using the formal written method of long multiplication.

Multiply one-digit numbers with up to two decimal places by whole numbers.

Children in Year 6 are expected to use the short or long multiplication methods in end of year tests.

ThHTO x O
(Short multiplication – multiplication by a single digit as Y5 to reinforce understanding)

4346 x 8

x	4000	300	40	6
8	32000	2400	320	48

$$\begin{array}{r} 32000 \\ +\ 2400 \\ +\ 320 \\ +\ 48 \\ \hline 34768 \\ \hline \end{array}$$

4.92 x 3

x	4	0.9	0.02
3	12	2.7	0.06

$$\begin{array}{r} 12.00 \\ +\ 0.70 \\ +\ 0.06 \\ \hline 12.76 \\ \hline \end{array}$$

Children should progress onto the short form TOxO algorithm as a written method as illustrated below:

Short multiplication

24 × 6 becomes

$$\begin{array}{rrr} & 2 & 4 \\ \times & & 6 \\ \hline 1 & 4 & 4 \\ \hline & {}_2 & \end{array}$$

Answer: 144

342 × 7 becomes

$$\begin{array}{rrrr} & 3 & 4 & 2 \\ \times & & & 7 \\ \hline 2 & 3 & 9 & 4 \\ \hline & {}_2 & {}_1 & \end{array}$$

Answer: 2394

2741 × 6 becomes

$$\begin{array}{rrrrr} & 2 & 7 & 4 & 1 \\ \times & & & & 6 \\ \hline 1 & 6 & 4 & 4 & 6 \\ \hline & {}_4 & {}_2 & & \end{array}$$

Answer: 16 446

TO x TO (Long multiplication – multiplication by more than a single digit to reinforce understanding)

72 x 38

x	70	2
30	2100	60
8	560	16

```
   2100
+   560
+    60
+    16
   2736
    1
```

```
   72
 X 38
   16  (8x2)
  560  (8x70)
   60  (30x2)
 2100  (30x70)
 2736
  1
```

Children are expected to use the formal algorithm (HTOxTO) as a long formal written method as illustrated below:

<u>Long multiplication</u>

124 × 26 becomes

```
     1  2
     1  2  4
 ×      2  6
 -----------
     7  4  4
  2  4  8  0
 -----------
  3  2  2  4
 -----------
  1  1
```

Answer: 3224

Here the carrying for multiplication are shown at the top.

Resources: whiteboards, ITP

Key Vocabulary: product, multiplication, lots of, groups of, multiply, x, multiply by, multiple of, once, twice , three times etc, repeated addition array, row, column, double, group in pairs, threes, tens etc