PROGRESSION THROUGH CALCULATIONS FOR DIVISION

We acknowledge Lancashire County Council as the main source for this document with some amendments specific to the Horwich and Blackrod Cluster.

It is imperative that children gain concrete experiences through play and manipulation before moving to any structured work or method. This is essential when any new concept is introduced.

- These standards are age-related expectations and therefore we expect the majority of children to achieve them.
- New learning is likely to be taught to groups rather than the whole class to acknowledge the different learning stages of the children.
- Children need to understand that division can be sharing or grouping (repeated subtraction).
- Children should understand that division is the inverse of multiplication.
- Children should understand that, unlike multiplication, division is **not** commutative. Also that it not associative i.e. 30 ÷ (5 ÷ 2) is not the same as (30 ÷ 5) ÷ 2
- Ensure that children understand the = sign means is the same as, not makes, and that children see calculations where the equals sign is in a different position, e.g. 12 ÷ 3 = 4 and 4 = 12 ÷ 3.
- Children should be encouraged to approximate before calculating and check whether their answer is reasonable.

By the end of year 6, children will have a range of calculation methods, mental and written. Selection will depend upon the numbers involved.

Children should not be made to go onto the next stage if:

1) **they are not ready.**
2) **they are not confident.**

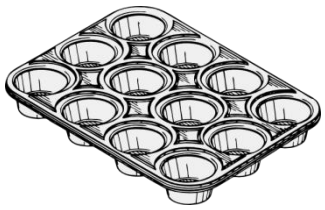
Children should be encouraged to consider if a mental calculation would be appropriate before using written methods.

Reception

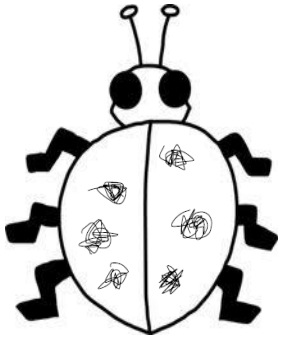
Early Learning Goal:
Children solve problems, including halving and sharing.

Children are encouraged to develop a mental picture of the number system in their heads to use for calculation. They should experience practical calculation opportunities involving equal groups and sharing items using a wide variety of equipment, including small world play, role play, counters, cubes etc.

Children may also investigate sharing items or putting items into groups using items such as egg boxes, ice cube trays and baking tins which are arrays.



They may develop ways of recording calculations using pictures, etc.



A child's jotting showing halving six spots between two sides of a ladybird.

A child's jotting showing how they shared the apples at snack time between two groups.

Resources:
Small world play, role play, counters, cubes, Numicon, models and images, ITP's etc

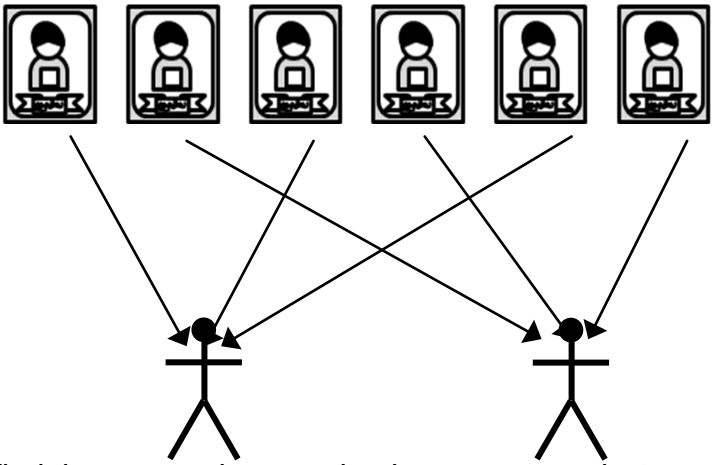
Key Vocabulary:
Sharing and grouping

Y1

End of Year Objective:
Solve one-step problems involving division by calculating the answer using concrete objects, pictorial representations and arrays with the support of the teacher.

In year one, children will continue to solve division problems using practical equipment and jottings. They should use the equipment to share objects and separate them into groups, answering questions such as

6 football stickers are shared between 2 people, how many do they each get?
Children may solve this by using a 'one for you, one for me' strategy until all of the cards have been given out.



Children should find the answer by counting how many cards **1 person** has got.

There are 6 football stickers, how many people can have 2 stickers each?

Children should find the answer by counting how many **groups of 2** there are.

The teacher can model the link between sharing and grouping in the following way by relating back to the first football sticker question:

Placing the football stickers in a bag or box, the teacher can ask the children how many stickers would need to be taken out of the box to give each person one sticker each (i.e. 2) and exemplify this by putting the cards in groups of 2 until all cards have been removed from the bag.

Children should be introduced to the concept of simple remainders in their calculations at this practical stage, being able to identify that the groups are not equal and should refer to the remainder as '… left over'.

Resources:
Small world play, role play, counters, cubes, Numicon, arrays, models and images, ITP's etc

Key Vocabulary:
Sharing, grouping and arrays

Y2

End of Year Objective:
Calculate mathematical statements for division within the multiplication tables and write them using the division (÷) and equals (=) signs.

Children will utilise practical equipment to represent division calculations as grouping (repeated subtraction) and use jottings to support their calculation, e.g.

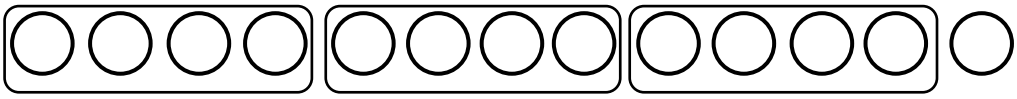
12 ÷ 3 =

Children need to understand that this calculation reads as 'How many groups of 3 are there in 12?' or '12 divided into groups of 3.'

This will be linked closely to work on arrays to support work in multiplication.

Children should also move onto calculations involving remainders.

13 ÷ 4 =



13 ÷ 4 = 3 remainder 1
Children need to be able to make decisions about what to do with remainders after division and round up or down accordingly. In the calculation 13 ÷ 4, the answer is 3 remainder 1, but whether the answer should be rounded up to 4 or rounded down to 3 depends on the context, as in the examples below:

I have £13. Books are £4 each. How many can I buy?
Answer: 3 (the remaining £1 is not enough to buy another book)

Apples are packed into boxes of 4. There are 13 apples. How many boxes are needed?
Answer: 4 (the remaining 1 apple still needs to be placed into a box)

Resources:
Role play, counters, cubes, Numicon, arrays, models and images, ITP's etc

Key Vocabulary:
Sharing, grouping, division, remainders and arrays

Y3

End of Year Objective:
Write and calculate mathematical statements for division using the multiplication tables that they know, including for two-digit numbers divided by one-digit numbers, progressing to formal written methods.*

**Although the objective suggests that children should be using formal written methods, the National Curriculum document states "The programmes of study for mathematics are set out year-by-year for key stages 1 and 2. Schools are, however, only required to teach the relevant programme of study by the end of the key stage. Within each key stage, schools therefore have the flexibility to introduce content earlier or later than set out in the programme of study." p4*

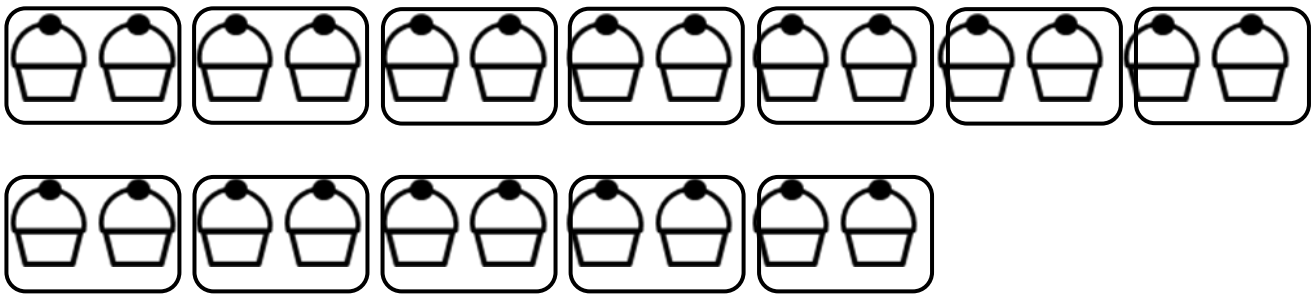
It is more beneficial for children's understanding to go through the expanded methods of calculation as steps of development towards a formal written method.

Children will continue to use grouping (repeated subtraction) to represent their calculations, answering questions such as:

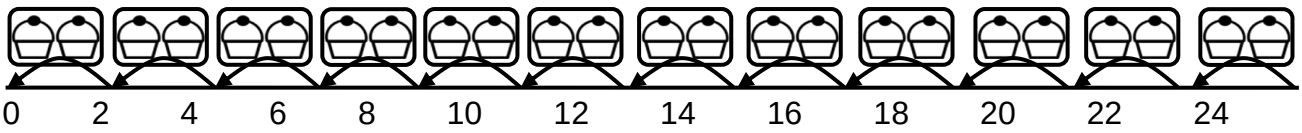
24 ÷ 2 =

or

There are 24 cupcakes, how many people can have 2 cupcakes each?



This should also be modelled alongside a number line to emphasise that grouping is repeated subtraction. Although number lines assist with mental calculations overlaps and links should always be made to aid understanding



Children need to be able to decide what to do with remainders after division and round up or down accordingly. They should make sensible decisions about rounding up or down after division. For example 62 ÷ 8 is 7 remainder 6, but whether the answer should be rounded up to 8 or rounded down to 7 depends on the context.

e.g. I have 62p. Sweets are 8p each. How many can I buy?
Answer: 7 (the remaining 6p is not enough to buy another sweet)

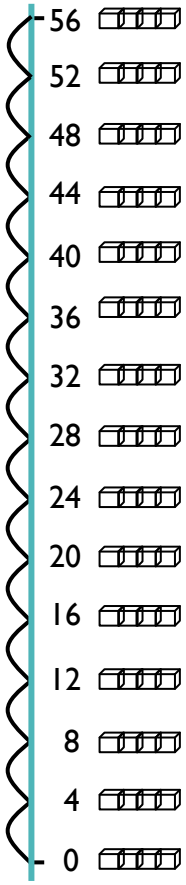
Apples are packed into boxes of 8. There are 62 apples. How many boxes are needed?
Answer: 8 (the remaining 6 apples still need to be placed into a box)

In preparation for developing the 'chunking' method of division, children should first use the repeated subtraction on a vertical number line alongside the continued use of practical equipment. There are two stages to this:

Stage 1 – repeatedly subtracting individual groups of the divisor
Stage 2 – subtracting multiples of the divisor (initially 10 groups and individual groups, then 10 groups and other multiples in line with tables knowledge)

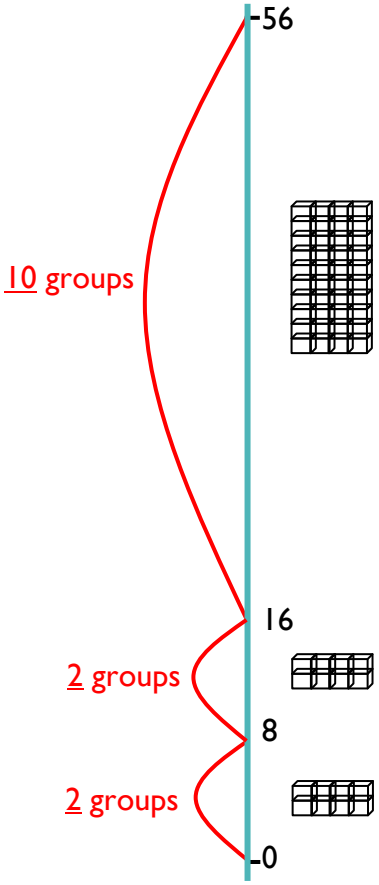
After each group has been subtracted, children should consider how many are left to enable them to identify the amount remaining on the number line. Use the number line both vertically and horizontally.

Stage 1
56 ÷ 4 = 14 (groups of 4)
of 4)



Stage 2
56 ÷ 4 = 10(groups of 4) + 2(groups of 4) + 2(groups

= 14(groups of 4)



Children should be able to solve real life problems including those with money and measures. They need to be able to make decisions about what to do with remainders after division and round up or down accordingly.

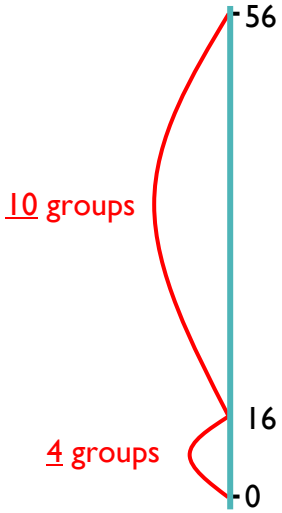
Resources:
Counters, cubes, Numicon, arrays, real life problems, models and images, ITP's etc
Key Vocabulary:
Sharing, grouping, division, remainders arrays, repeated subtraction, round up and round down

Y4

End of Year Objective:
Divide numbers up to 3 digits by a one-digit number using the formal written method of short division and interpret remainders appropriately for the context.

Children will continue to work on the vertical number line as introduced in Year 3.
When children are ready, and efficient in this method, they will progress on to the following.

Children will continue to develop their use of grouping (repeated subtraction) to be able to subtract multiples of the divisor, moving on to the use of the 'chunking' method.




```
   14
4 ) 56
  - 40      10x
   16
  - 16      4x
    0
```

Answer: 14

Children should write their answer above the calculation to make it easy for them and the teacher to distinguish.

The number line method used in year 3 can be linked to the chunking method to enable children to make links in their understanding.

Children will develop their use of grouping (repeated subtraction) to be able to subtract multiples of the divisor, developing the use of the 'chunking' method.

Long division (TU ÷ O)

72 ÷ 3

```
   24
3 ) 72
  - 30      10x
   42
  - 30      10x
   12
  -  6       2x
    6
  -  6       2x
    0
```

Answer : 24

1x	3
2x	6
4 x	12
10x	30
5x	15

Children should write **key facts** in a menu box. This will help them in identifying the largest group they can subtract in one chunk.

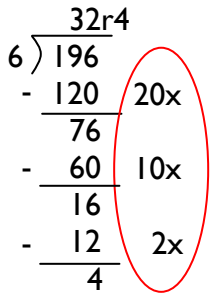
KEY FACTS
The 1,2,4,10,5 method links mental agility with doubling and links with the repeated addition method of multiplication

Children should write their answer above the calculation to make it easy for them and the teacher to distinguish.

Any remainders should be shown as integers, i.e. 14 remainder 2 or 14 r 2.

By the end of year 4, children should be able to use the chunking method to divide a three digit number by a single digit number. To make this method more efficient, the key facts in the menu box should be extended to include 4x and 20x, e.g.

196 ÷ 6



1x	3
2x	6
4 x	12
10x	30
5x	15

Key facts box

1x	6
2x	12
4x	24
5x	30
10x	60
20x	120

Children should be able to solve real life problems including those with money and measures. They need to be able to make decisions about what to do with remainders after division and round up or down accordingly.

e.g. There are 320 people travelling to Chester, each coach can hold 50 people. How many coaches need to be booked?
Answer: 7 (the remaining 20 people also need a method of transport)

Resources:
Numicon, arrays, real life problems, vertical number lines, models and images, ITP's etc

Key Vocabulary:
Sharing, grouping, division, remainders, arrays, repeated subtraction, round up, round down, chunking, groups of, divisors, multiples, integers and long division

Y5

End of Year Objective:
Divide numbers up to 4 digits by a one-digit number using the formal written method of short division and interpret remainders appropriately for the context.

Children can start to subtract larger multiples of the divisor (e.g. 20x).

Long division (HTO ÷ O)

196 ÷ 6

```
      32 r 4
6 ) 196
  - 180      30x6
    16
  -  12       2x6
     4
```

1x	6
2x	12
4x	24
5x	30
10x	60
20x	120

The key facts in the menu box should be extended to include 4x and 20x.

Answer : 32 remainder 4 or 32 r 4

Any remainders should be shown as integers, i.e. 14 remainder 2 or 14 r 2.

By the end of year 5, children should be able to use the chunking method to divide a four digit number by a single digit number. If children still need to use the key facts box for facts within their multiplication tables, it can be extended to include 100x.

2458 ÷ 7

```
     351 r 1
7 ) 2458
  - 2100    300x
    358
  -  350     50x
     8
  -   7      1x
     1
```

Children when mathematically ready move onto the short division method.

Short division (HTU ÷ O)

98 ÷ 7 becomes

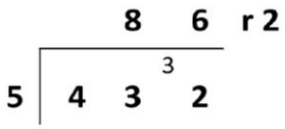
```
    1  4
   ----
7 | 9  8
      2
```

Answer: 14

Children need to be able to decide what to do after division and round up or down accordingly. They should make sensible decisions about rounding up or down after division.

For example 240 ÷ 52 is 4 remainder 32, but whether the answer should be rounded up to 5 or rounded down to 4 depends on the context. (See Year 4 example)

432 ÷ 5 becomes



Answer: 86 remainder 2

Children should be provided with practical opportunities to realise you cannot break the hundred up, you need to exchange. They use base10 equipment to embed this.

Children should be able to solve real life problems including those with money and measures.

Resources:
Real life problems, models and images, ITP's, base 10 equipment etc

Key Vocabulary:
Sharing, grouping, division, remainders, arrays, repeated subtraction, round up, round down, chunking, groups of, divisors, multiples, integers, long division and short division

Y6

End of Year Objective:
Divide numbers up to 4 digits by a two-digit number using the formal written method of short division where appropriate, interpreting remainders according to the context.

Use written division methods in cases where the answer has up to two decimal places.

Long division

To develop the chunking method further, it should be extended to include dividing a four-digit number by a two-digit number, e.g.

6367 ÷ 28

```
   227r11
28)6367
 - 5600   200x
   767
 - 560    20x
   207
 - 140    5x
   67
 - 56     2x
   11
```

Children may still use the menu box if required, but would also be expected to use larger multiples of the divisor (e.g. 20x, 30x, 40x).

1x	28
2x	48
4x	96
5x	140
10x	280
20x	480

10x	280
20x	560
40x	1120
50x	1400
100x	2800
200x	5600

Short division (HTO÷O)

Any remainders should be shown as fractions, i.e. if the children were dividing 432 by 5, the answer should be shown as 86 $^{2}/_{5}$.

432 ÷ 5 becomes

```
     8  6  r2
   ┌──────
5  │ 4  3  2
      3
```

Answer : 86 $^{2}/_{5}$

Note:
(HTO÷ O) – Use short division

This method should be extended to be used with decimals with up to two decimal places.

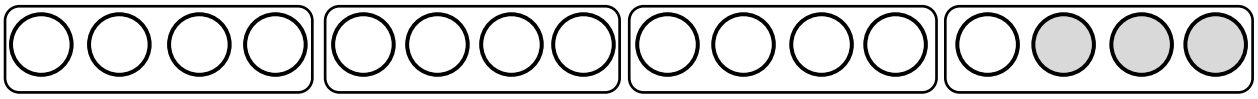
e.g. 950 ÷ 4

$$4\overline{)9^{1}5^{3}0.^{2}0}\quad 237.5$$

Children should be able to solve real life problems including those with money and measures. They need to be able to make decisions about what to do with remainders after division and round up or down accordingly.

In addition, children should also be able to use the chunking method and solve calculations interpreting the remainder as a decimal up to two decimal places.

For simple fraction and decimal equivalents, this could also be demonstrated using a simple calculation such as 13 ÷ 4 to show the remainder initially as a fraction.



Using practical equipment, children can see that for 13 ÷ 4, the answer is 3 remainder 1, or put another way, there are three whole groups and a remainder of 1. This remainder is one part towards a full group of 4, so is $\frac{1}{4}$. To show the remainder as a fraction, it becomes the numerator where the denominator is the divisor (the number that you are dividing by in the calculation).

3574 ÷ 8

8 ⟌ 3574		
- 3200	400x8	
374		
- 320	40x8	
54		
- 48	6x8	
6		

6 ← remainder

8 ← divisor

So 3574 ÷ 8 is $446\frac{6}{8}$

(when the remainder is shown as a fraction)

To show the remainder as a decimal relies upon children's knowledge of decimal fraction equivalents. For decimals with no more than 2 decimal places, they should be able to identify:

Half: $\frac{1}{2} = 0.5$

Quarters: $\frac{1}{4} = 0.25$, $\frac{3}{4} = 0.75$

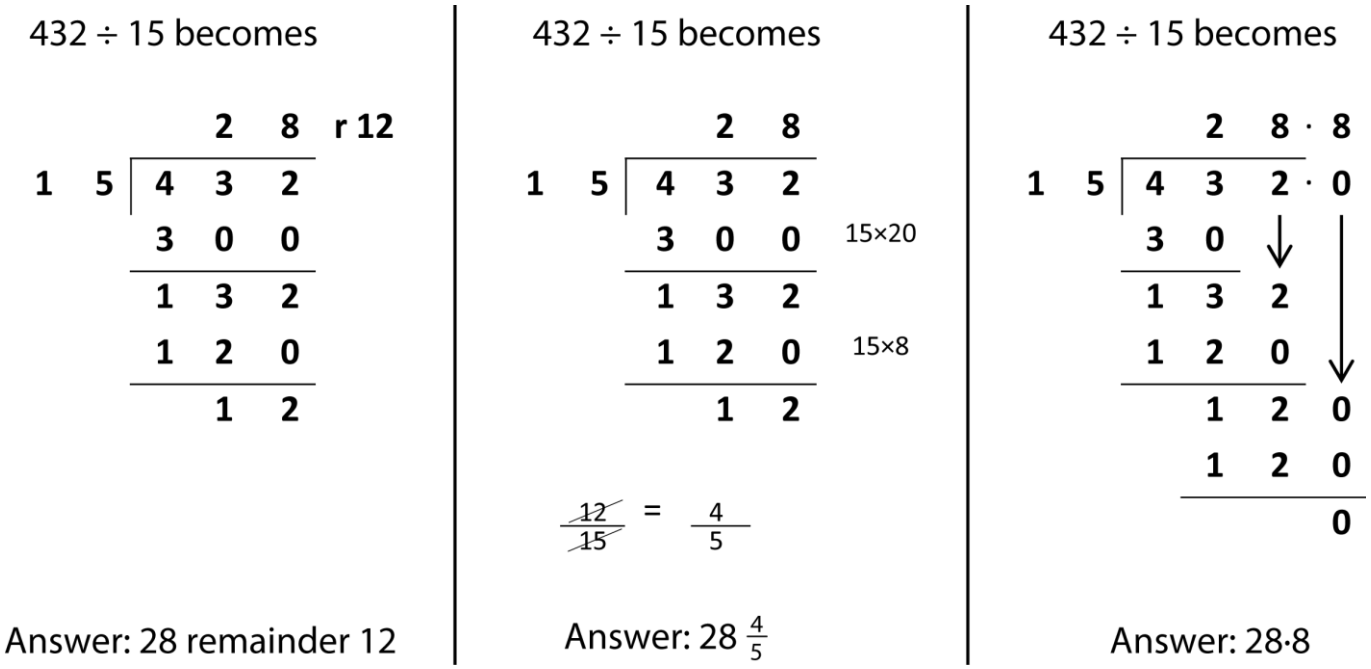
Fifths: $\frac{1}{5} = 0.2$, $\frac{2}{5} = 0.4$, $\frac{3}{5} = 0.6$, $\frac{4}{5} = 0.8$

Tenths: $\frac{1}{10} = 0.1$, $\frac{2}{10} = 0.2$, $\frac{3}{10} = 0.3$, $\frac{4}{10} = 0.4$, $\frac{5}{10} = 0.5$, $\frac{6}{10} = 0.6$, $\frac{7}{10} = 0.7$, $\frac{8}{10} = 0.8$, $\frac{9}{10} = 0.9$

and reduce other equivalent fractions to their lowest terms.

In the example above, 3574 ÷ 8, children should be able to identify that the remainder as a fraction of $\frac{6}{8}$ can be written as $\frac{3}{4}$ in its lowest terms. As $\frac{3}{4}$ is equivalent to 0.75, the answer can therefore be written as 446.75

LONG DIVISION



Resources:
Real life problems, models and images, ITP's, base 10 equipment etc
Key Vocabulary:
Sharing, grouping, division, remainders arrays, repeated subtraction, round up, round down, decimal number, decimal point, long division and short division